EMLV
BUSINESS SCHOOL
DE VINCI PARIS

Financial econometrics

Asset returns and statistical concepts

1. (Holding period) return on an asset

- There are **two sources of return** for any investment :
  - **Income** : cash that investors periodically receive as a result of owning an investment (dividends, coupons…)
  - **Capital gain (or loss) :** the difference between the sell price and the purchase price
- (**Holding period**) **return** R_t is the **total return received from holding an asset** or portfolio of assets over a **period of time *t***.

$$R_t = \frac{\text{Income} + \text{Capital gain (or loss)}}{\text{Initial Investment}}$$

1. (Holding period) return on an asset

The **(holding period) return on a stock** over a period of time *t* :

$$R_t = \frac{Div_t}{P_{t-1}} + \frac{P_t - P_{t-1}}{P_{t-1}}$$

*The return on a stock = dividend yield + Return from **capital gain** or loss*

The **(holding period) return on a bond** over a period of time *t* :

$$R_t = \frac{coupon_t}{P_{t-1}} + \frac{P_t - P_{t-1}}{P_{t-1}}$$

*The return on a bond = Return from **coupons** + Return from **capital gain** or loss*

Dividends: These are payments made by a company to its shareholders, typically from its profits

Coupons: These are interest payments made to bondholders by the issuer of the bond.

1. (Holding period) return on an asset

Exercise 1 :

Calculate a **one-year holding period return** for the following four investment alternatives.

	savings account	Stock	Bond	Zero coupon bond
Income (in €)				
1st quarter	15 €	10 €	0 €	0 €
2nd quarter	15 €	0 €	70 €	0 €
3rd quarter	15 €	0 €	0 €	0 €
4th quarter	15 €	15 €	70 €	0 €
Investment Value (in €)				
At the beginning of the year	1000 €	1900 €	970 €	3300 €
At the end of the year	1000 €	2000 €	1000 €	3000 €

Which investment would you prefer, assuming they are of equal risk?

Diapo 9 - Solution

Given Information:

- Initial investment at the beginning of the year: 1,000 euros
- Ending balance at the end of the year: 1,000 euros
- Quarterly income: 15 euros (received 4 times a year)

Calculate the Total Income Received : Since income is received quarterly, and each quarter provides 15 euros. Then

$$\text{Total Income} = 15 \text{ euros} \times 4 \text{ quarters} = 60 \text{ euros}$$

Calculate the Total Return: The total return includes both the income received and the change in the value of the initial investment

$$\text{Total Return} = \frac{\text{Income received} + (\text{Ending balance} - \text{Initial investment})}{\text{Initial investment}}$$

Plugging in the values:

$$\text{Total Return} = \frac{60 + (1000 - 1000)}{1000} = 0{,}06 \text{ or } 6\%$$

Conclusion: The return on saving money in the account over the year is 6%. This return comes solely from the quarterly income received, as the ending balance was the same as the initial investment

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PLAN

1. **(Holding period) return** on an asset
2. **Risk-free** vs **risky assets**
3. **Historical average** (a simple measure of **expected return**)
4. **Variance** and **standard deviation :** **(**Simple measures of **risk)**
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6. **Excel application**

2. Risk-free vs risky assets

Risk-free asset vs risky asset :

- A **risk-free asset** is an asset whose **returns in the future are known with certainty today**.
  - Overall, government securities are considered as risk-free
  - Examples : **T-bills** (US), les Obligations Assimilables du Trésor (**OAT**, France), "**Bunds**" in Germany…
  - The rate of interest earned from investing funds in a government securitiy is called the **risk-free rate**, and denoted R_f (Risk free)

2. Risk-free vs risky assets

Risk-free asset vs risky asset :

- A **risky asset** is an asset with **uncertain future returns**.
  - Examples : Stocks, Coporate bonds, Currencies…
  - Risky assets are **expected to earn a higher average return** than **risk-free assets**
  - The **difference** between **the return of a risky asset R_i ,** and **the risk-free rate R_f is called** the risk premium : $R_i - R_f$

Asset Class Returns

2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	YTD
REIT 35.1%	EM 39.8%	HG Bnd 5.2%	EM 79.0%	REIT 28.0%	REIT 8.3%	REIT 19.7%	Sm Cap 38.8%	REIT 28.0%	REIT 2.8%	Sm Cap 21.3%	EM 37.8%	Cash 1.8%	Lg Cap 31.5%	Sm Cap 20.0%	REIT 21.4%
EM 32.6%	Int'l Stk 11.6%	Cash 1.7%	HY Bnd 57.5%	Sm Cap 26.9%	HG Bnd 7.8%	EM 18.6%	Lg Cap 32.4%	Lg Cap 13.7%	Lg Cap 1.4%	HY Bnd 17.5%	Int'l 25.6%	HG Bnd 0.0%	REIT 28.7%	EM 18.7%	Sm Cap 17.5%
Int'l Stk 26.9%	AA 7.6%	AA -22.4%	Int'l Stk 32.5%	EM 19.2%	HY Bnd 4.4%	Int'l Stk 17.9%	Int'l Stk 23.3%	AA 6.9%	HG Bnd 0.6%	Lg Cap 12.0%	Lg Cap 21.8%	HY Bnd -2.3%	Sm Cap 25.5%	Lg Cap 18.4%	Lg Cap 15.3%
Sm Cap 18.4%	HG Bnd 7.0%	HY Bnd -26.4%	REIT 28.0%	HY Bnd 15.2%	Lg Cap 2.1%	Sm Cap 16.4%	AA 11.5%	HG Bnd 6.0%	Cash 0.0%	EM 11.6%	Sm Cap 14.7%	REIT -4.0%	Int'l Stk 22.7%	AA 9.8%	Int'l 9.2%
AA 16.7%	Lg Cap 5.5%	Sm Cap -33.8%	Sm Cap 27.2%	Lg Cap 15.1%	AA 0.3%	Lg Cap 16.0%	HY Bnd 7.4%	Sm Cap 4.9%	Int'l Stk -0.4%	REIT 8.6%	AA 14.6%	Lg Cap -4.4%	AA 18.9%	Int'l Stk 8.3%	AA 7.7%
Lg Cap 15.8%	Cash 4.4%	Lg Cap -37.0%	Lg Cap 26.5%	AA 13.5%	Cash 0.1%	HY Bnd 15.6%	REIT 2.9%	HY Bnd 2.5%	AA -1.3%	AA 7.2%	REIT 8.7%	AA -5.6%	EM 18.9%	HY Bnd 7.5%	EM 7.6%
HY Bnd 11.8%	HY Bnd 2.2%	REIT -37.7%	AA 24.6%	Int'l Stk 8.2%	Sm Cap -4.2%	AA 12.2%	Cash 0.1%	Cash 0.0%	Sm Cap -4.4%	HG Bnd 2.7%	HY Bnd 7.5%	Sm Cap -11.0%	HY Bnd 14.4%	HG Bnd 6.1%	HY Bnd 3.7%
Cash 4.7%	Sm Cap -1.6%	Int'l Stk -43.1%	HG Bnd 5.9%	HG Bnd 6.5%	Int'l Stk -11.7%	HG Bnd 4.2%	HG Bnd -2.0%	EM -1.8%	HY Bnd -4.6%	Int'l Stk 1.5%	HG Bnd 3.5%	Int'l Stk -13.4%	HG Bnd 8.7%	Cash 0.6%	Cash 0.0%
HG Bnd 4.3%	REIT -15.7%	EM -53.2%	Cash 0.1%	Cash 0.1%	EM -18.2%	Cash 0.1%	EM -2.3%	Int'l Stk -4.5%	EM -14.6%	Cash 0.3%	Cash 0.8%	EM -14.3%	Cash 2.2%	REIT -5.1%	HG Bnd -1.6%

Abbr.	Asset Class – Index	Annual	Best	Worst
Lg Cap	Large Cap Stocks – S&P 500 Index	9.88%	32.4%	-37.0%
Sm Cap	Small Cap Stocks – Russell 2000 Index	8.91%	38.8%	-33.8%
Int'l Stk	International Developed Stocks – MSCI EAFE Index	4.97%	32.5%	-43.1%
EM	EM Stocks – MSCI Emerging Markets Index	6.95%	79.0%	-53.2%
REIT	REITs – FTSE NAREIT All Equity Index	7.15%	35.1%	-37.7%
HG Bnd	High Grade Bonds – Bloomberg Barclays U.S. Agg Index	4.40%	8.7%	-2.0%
HY Bnd	High Yield Bonds – ICE BofA US High Yield Index	7.44%	57.5%	-26.4%
Cash	Cash – S&P U.S. Treasury Bill 0-3 Mth Index	1.11%	4.7%	0.0%
AA	Asset Allocation Portfolio*	7.02%	24.6%	-22.4%

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When you make an investment, you usually start by gathering some data to help you estimate your potential return.
For example, if you buy an apartment with the intention of renting it out, you can get an idea of how much rent you might earn. And if you plan to sell it after a year, you can also estimate its resale value.
This kind of estimation gives you a sense of what's called the **expected return** — basically, how much you might gain from your investment.

More formally, we have the definition of the **expected return** which is the **rate of return** an investor can expect to earn by holding an asset over a period of time.

- A **simple measure** of the expected return is the **historical average return**

$$\hat{E}(R_i) = \frac{1}{T} \sum_{t=1}^{T} R_i(t)$$

3. Historical average
(a simple measure of expected return)

The **expected return** : The **rate of return** an investor can expect to earn by holding an asset over a period of time.

- A **simple measure** of the expected return is the **historical average return**

$$\hat{E}(R_i) = \frac{1}{T}\sum_{t=1}^{T} R_i(t)$$

3. Historical average
(a simple measure of expected return)

Exercise 2 :

The **historical returns** for two stocks—FB and FM—are summarized in the following table for the period 2019 to 2022 :

	2019	2020	2021	2022
FB	-10%	10%	5%	2%
FM	21%	21%	30%	-15%

- **Calculate the expected return** for each stock

Solution Exercice 2

To calculate the expected return for each stock, we use the formula for the arithmetic mean of the historical returns. The expected return is simply the average of the returns over the given period.

For Stock FB

$$E(R_{FB}) = \frac{-10\%+10\%+5\%+2\%}{4} = 1.75\%$$

For Stock FM

$$E(R_{FB}) = \frac{21\%+21\%+30\%-15\%}{4} = 14.25\%$$

Conclusion:

- The expected return for **Stock FB** is **1.75%**.
- The expected return for **Stock FM** is **14.25%**.

These expected returns represent the average performance of each stock over the period from 2019 to 2022.

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4. Variance and standard deviation (simple measures of risk)

- **Risk** is **the possibility** that the **realized return** from holding an asset **will deviate** from its **expected return**.

- **Variance** and **standard deviation** of returns are common measures of investment risk :

  - **The variance** of the return on a stock i :

$$\hat{\sigma}_i^2 = \frac{1}{T-1} \sum_{t=1}^{T} \left[R_i(t) - \hat{E}(R_i) \right]^2$$

  - **The standard deviation (volatility)** of the return on a stock i :

$$\sigma_i = \sqrt{\sigma_i^2}$$

High volatility (or variance) ➔ High dispersion ➔ High risk

Diapo 23 Explanation

It is true we talked a lot about Risk without given a formel definition of it. A Risk is the possibility that the realized return from holding an asset will deviate from its expected return.

Here where two mathematical notions comes into play to measure risk. Variance and standard deviation of returns are common measures of investment risk :

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Exercise 3 :

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	2019	2020	2021	2022
FB	-10%	10%	5%	2%
FM	21%	21%	30%	-15%

- Calculate **the variance** and **standard deviation** for each stock
- Which stock appears to **be more risky ?**

Solution Exercise 3

Remember the formula

$$\hat{\sigma}_i^2 = \frac{1}{T-1} \sum_{t=1}^{T} \left[R_i(t) - \hat{E}(R_i) \right]^2$$

Here R_I the return in each period, $\hat{\mathrm{E}}(R_I)$ the average expected return and T the number of period

For Stock FB:

- Expected Return = 1.75% (calculated previously)
- Calculating Variance:

$$\sigma^2 = \frac{(-10-1{,}75)^2+(10-1{,}75)^2+(5-1{,}75)^2+(2-1{,}75)^2}{4-1} = 72.25$$

For Stock FM:

- Expected Return = 14.25% (calculated previously)
- Calculating Variance:

$$\sigma^2 = 398.5833\%$$

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5. Covariance and correlation
(simple measures of dependencies)

- **The covariance** between two asset returns is the average of the products of their deviations from the mean :

$$\hat{\text{cov}}(R_i, R_j) = \frac{1}{T-1} \sum_{t=1}^{T} \left[R_i(t) - \hat{E}(R_i) \right] \left[R_j(t) - \hat{E}(R_j) \right]$$

- **The covariance measures co-movement** between the returns of two assets

  - Positive : returns move together
  - Negative : returns move inversely

Covariance and correlation :

- **Correlation** between two asset returns is defined as **the covariance divided by the standard deviations** of the two asset returns:

$$corr(R_i, R_j) = \frac{cov(R_i, R_j)}{\sigma_i \sigma_j}$$

- **Correlation** is a **standardized version of covariance:**

$$-1 \leq corr(R_i, R_j) \leq 1$$

5. Covariance and correlation
(simple measures of dependencies)

- **Correlation** has no units. It is a pure measure of the co-movement of the two assets' returns and is bounded by −1 and +1

  - **Positive**: Returns move together

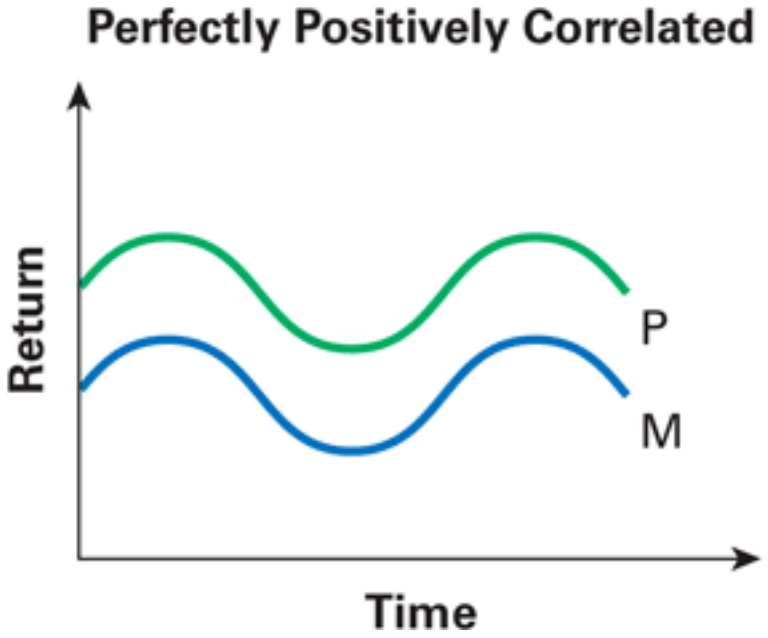
    Correlation = +1 ➔ Perfectly positively correlated

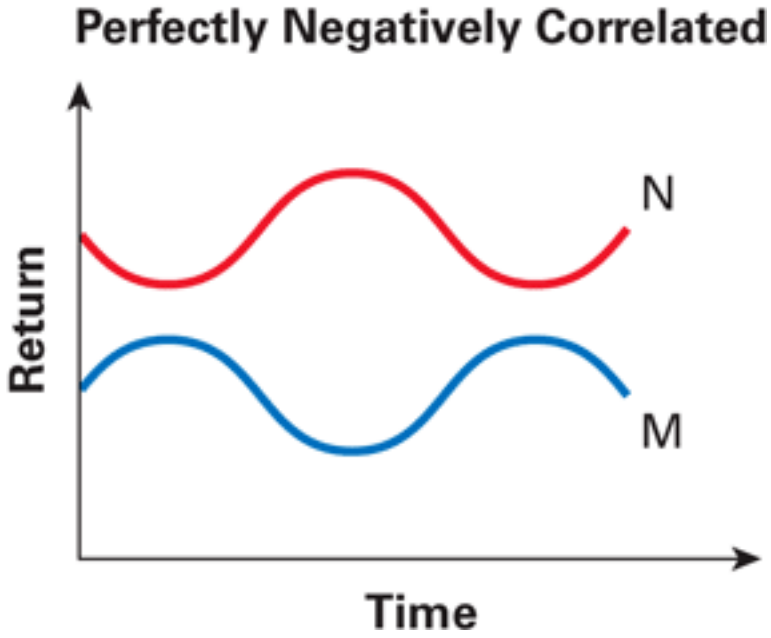
  - **Negative**: Returns move inversely

    Correlation = −1 ➔ Perfectly negatively correlated

  - **zero**: no linear relationship between the two assets

The Correlations of Returns between Investments M and P and Investments M and N





Exercise 4 :

The **historical returns** for two stocks—FB and FM—are summarized in the following table for the period 2019 to 2022 :

	2019	2020	2021	2022
FB	-10%	10%	5%	2%
FM	21%	21%	30%	-15%

- Calculate **the covariance** and **the correlation** between FB and FM returns

Solution Exercice 4

Covariance between FB and FM: 6.75
Correlation between FB and FM: 0.041

This correlation value indicates a very weak positive relationship between the returns of Stock FB and Stock FM, suggesting that their returns do not move together in a significant way.

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Excel application :

“Excel_application 1” contains monthly adjusted prices for the **CAC 40 index** and four stocks, namely, **L'Oréal**, **Vinci, Peugeot**, and **Kering** between 31/12/2012 and 31/12/2017

- Compute monthly returns for the CAC 40 index and the four stocks
- Compute the mean, variance, and standard deviation for the CAC 40 index and the four stocks
- Compute the covariance and the correlation between the market index and each of the four stocks